ME 243

Mechanics of Solids

Lecture 5: Beam: Shear and Moment

Ahmad Shahedi Shakil

Lecturer, Dept. of Mechanical Engg, BUET

E-mail: sshakil@me.buet.ac.bd, shakil6791@gmail.com

Website: teacher.buet.ac.bd/sshakil

Beam

- Beam is a member that bends due to application of transverse load.
- It supports load perpendicular to the axis applied at various points.
- Beams may be both straight and curves.
- Beams may have various types of cross-sections, e.g. square, rectangular, I-section, circular etc.

Classifications of straight beam

- Mainly 2 types of beam:

1. Statically determinate beam: The values of reactions can be obtained by using the conditions of static equilibrium. It can be classified as:

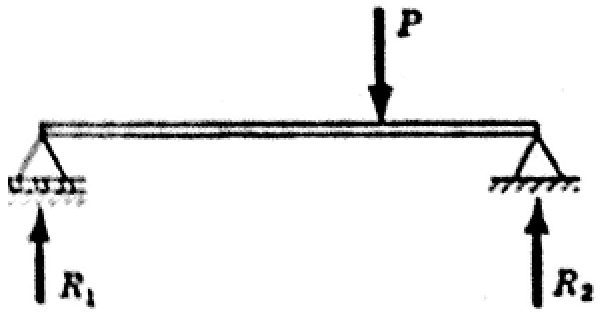
   a) Simply supported beam
   b) Cantilever beam
   c) Overhanging beam

2. Statically indeterminate beam: The values of reactions can not be obtained by using the conditions of static equilibrium only. It is of three types:

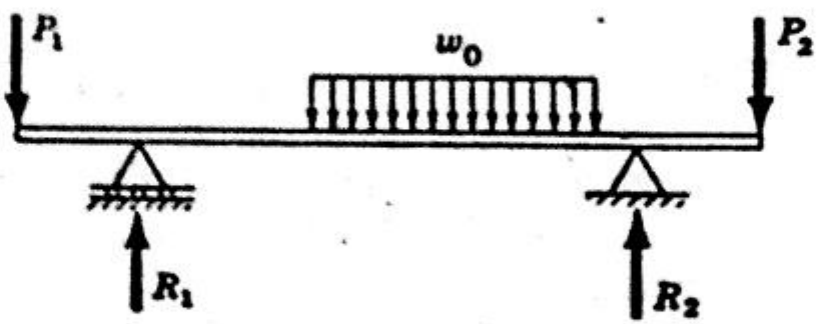
   a) Continuous beam
   b) Fixed beam
   c) Propped beam

Classifications of straight beam

- Simply supported beam: it is a beam with supports at the ends either by rollers or pins.

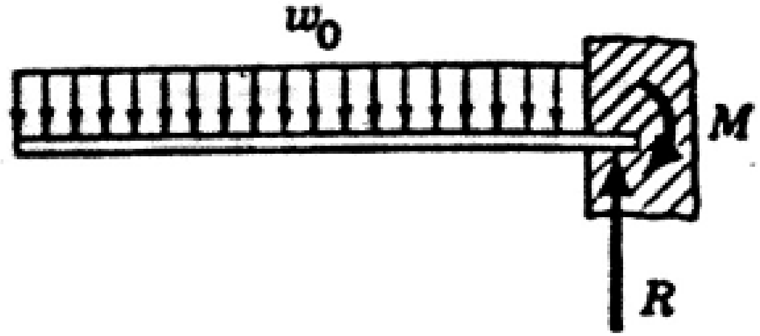


- Overhanging beam: It is a beam where one or both supports are not at the ends.

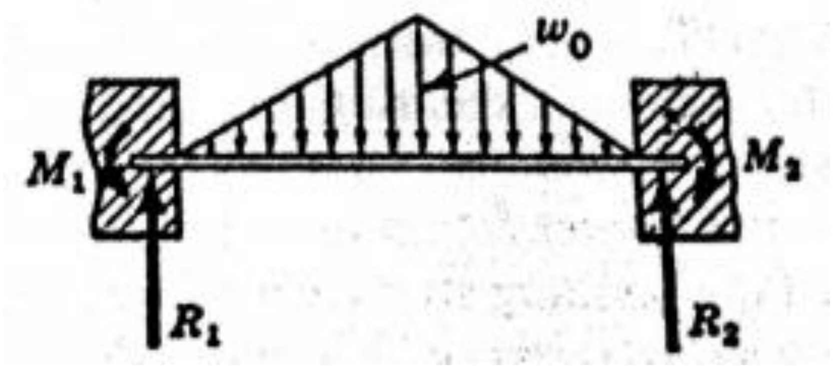


Classifications of straight beam

- Cantilever beam: It is a beam fixed at one end and free at other end.

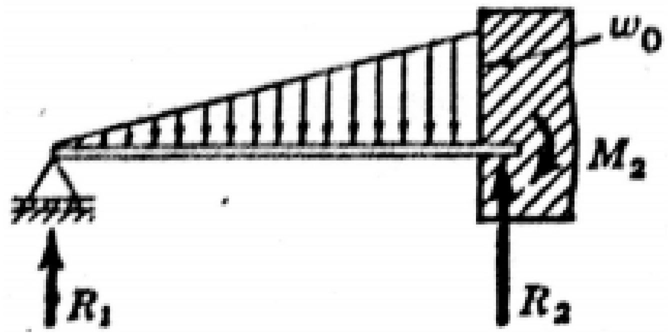


- Fixed beam: It is a beam fixed at both ends.

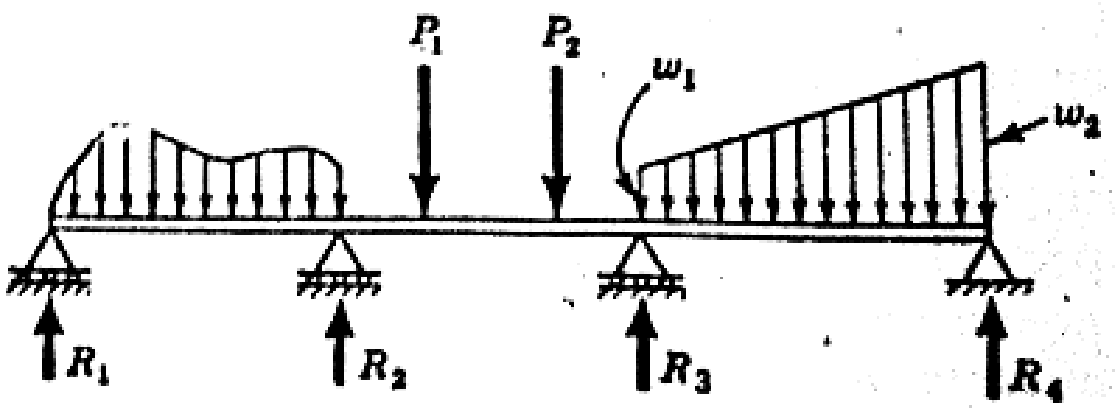


Classifications of straight beam

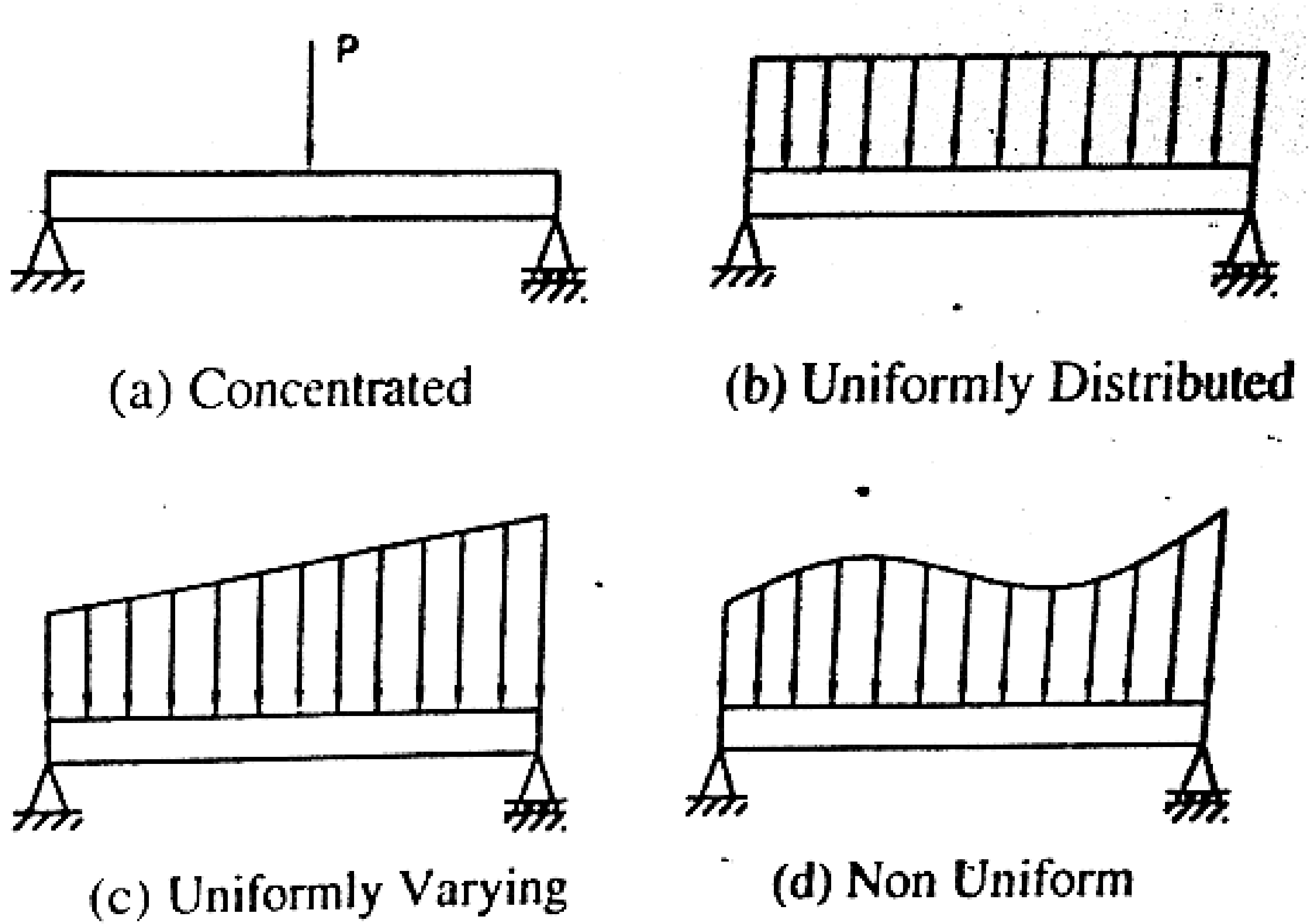
- Propped beam: It is beam fixed at one end and simply supported at other end.



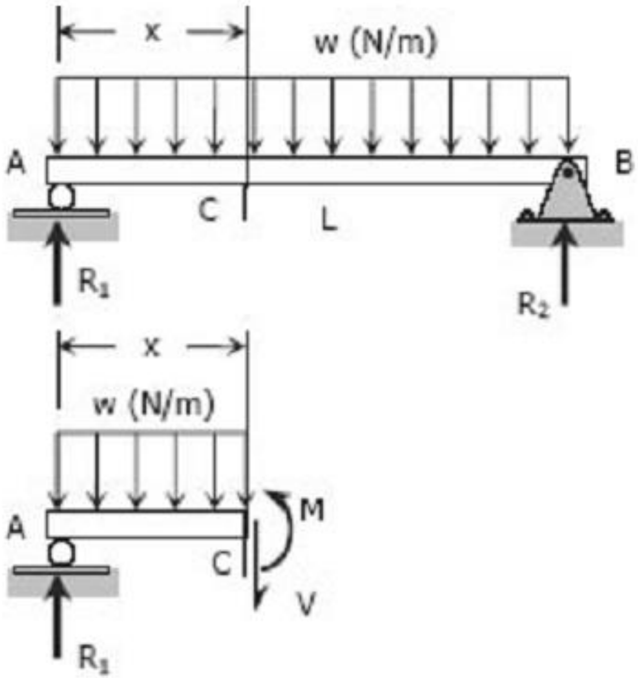
- Continuous beam: It is a beam that consists of intermediate supports.



Types of loads in beam



Shear and moment

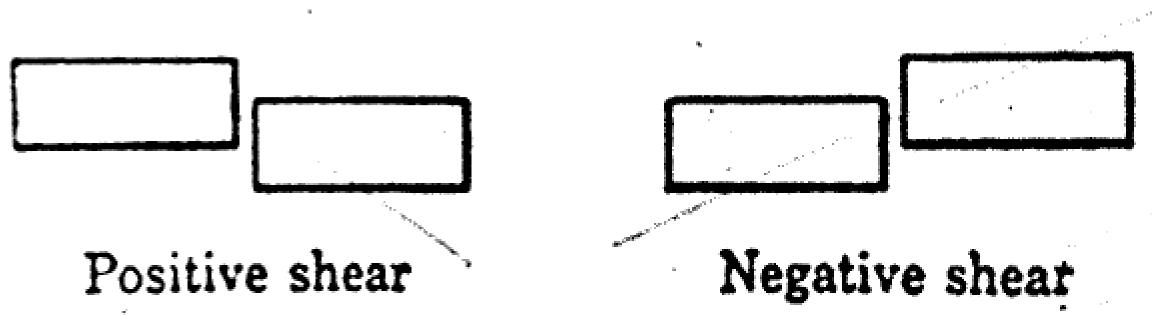


- Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R_1 and R_2.
- Assume that the beam is cut at point distance of x from he left support and the portion of the beam to the right of C be removed.
- The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the action of $R_1$and w_x.
- The couple M is called the resisting moment or bending moment and the force V is called the resisting shear or shear force.

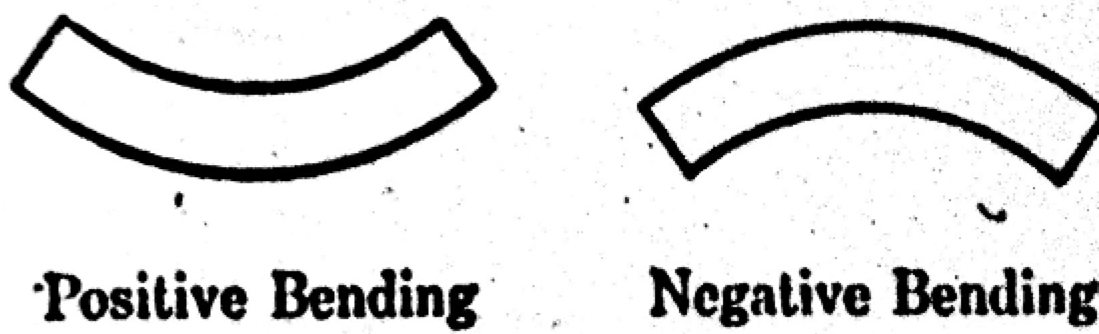
Shear and moment

- Sign convention:

Shear Force :

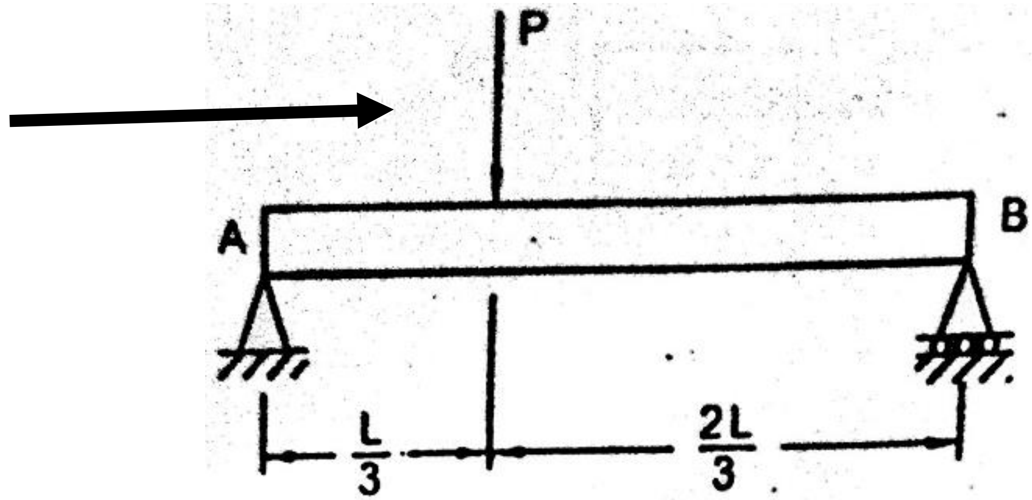


Bending moment :

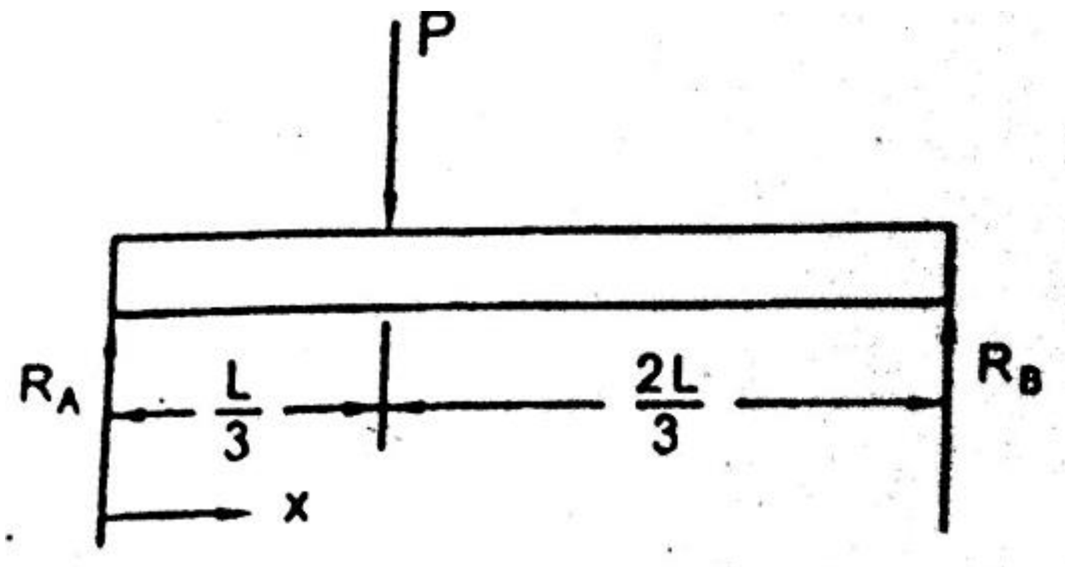


Shear and bending moment diagram

Concentrated load



Free body diagram of the entire beam is shown below.



Shear and bending moment diagram

In static equilibrium condition,

$$+\uparrow\sum F_y = 0$$

or, $R_A + R_B - P = 0$

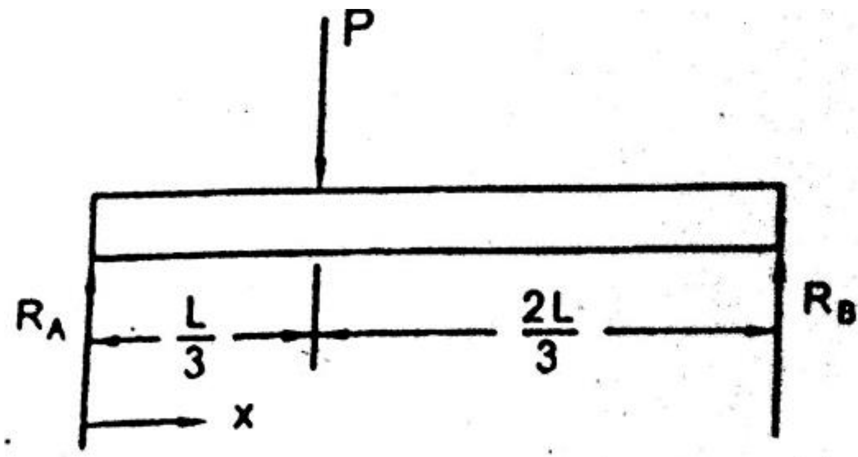
or, $R_A + R_B = P$

$$+\circlearrowleft\sum M_A = 0$$

$$R_B \times L - \frac{P \times L}{3} = 0$$

or, $R_B = \frac{P}{3}$

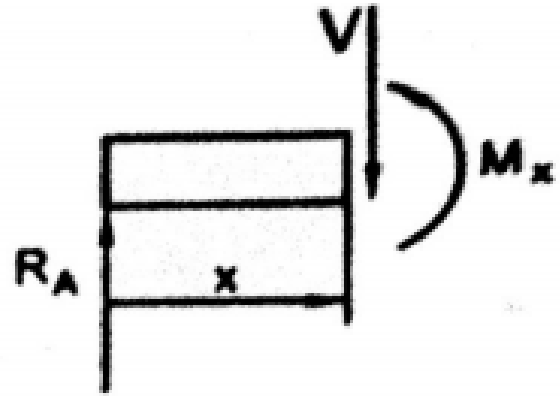
$$R_A = P - R_B = P - \frac{P}{3} = \frac{2P}{3}$$



Shear and bending moment diagram

Free body diagram of a section of the beam at the left of the concentrated load is considered as follows.

For $0 < x < L/3$



$$V_x = R_A = \frac{2P}{3}$$

$$M_x = R_A x$$

$$= \frac{2Px}{3}$$

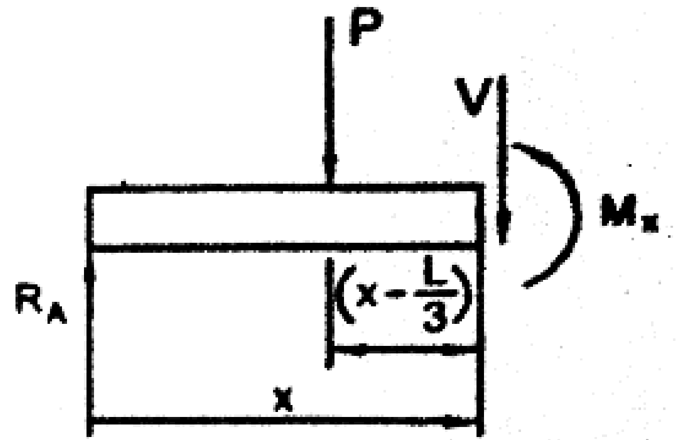
At $x = L/3$,

$$M_{x=L/3} = \frac{2PL}{9}$$

Shear and bending moment diagram

Free body diagram of a section of the beam at the right of the concentrated load is considered as shown in the following figure.

For $L/3 < x < L$



$$V_x = R_A - P = \frac{2P}{3} - P = -\frac{P}{3}$$

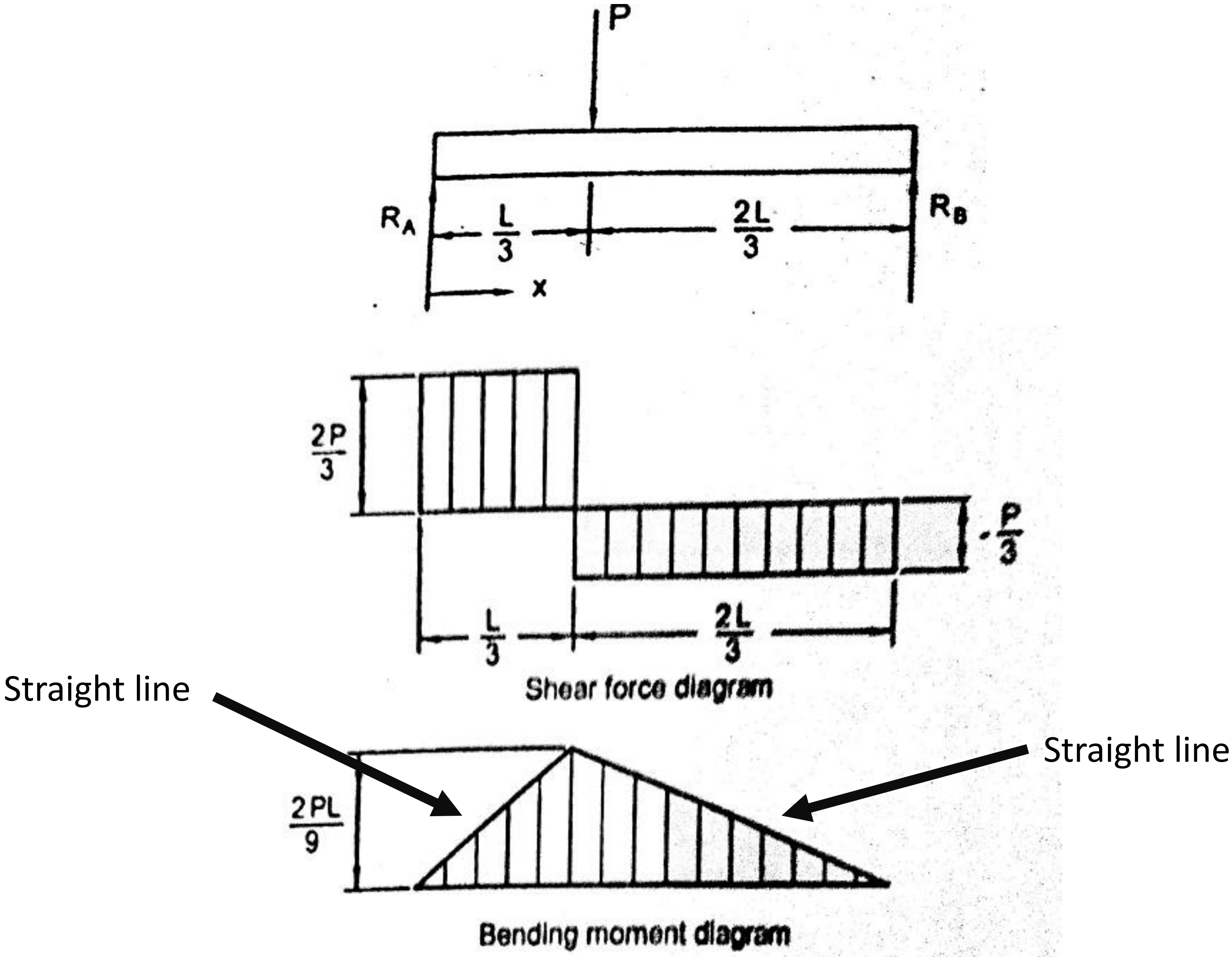
$$M_x = R_A x - P\left(x - \frac{L}{3}\right)$$

$$= \frac{2Px}{3} - Px + \frac{PL}{3}$$

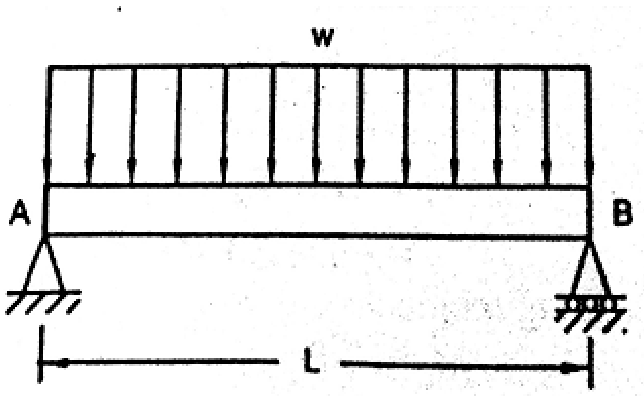
$$= -\frac{Px}{3} + \frac{PL}{3}$$

At $x = L/3$ $\qquad M_{x=L/3} = -\frac{PL}{9} + \frac{PL}{3} = \frac{2PL}{9}$

Shear and bending moment diagram

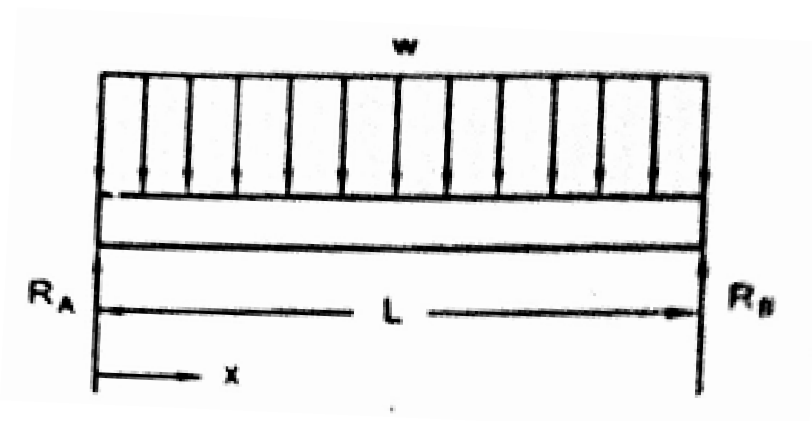


Shear and bending moment diagram



Uniformly distributed load

Free body diagram of the entire beam is given below.

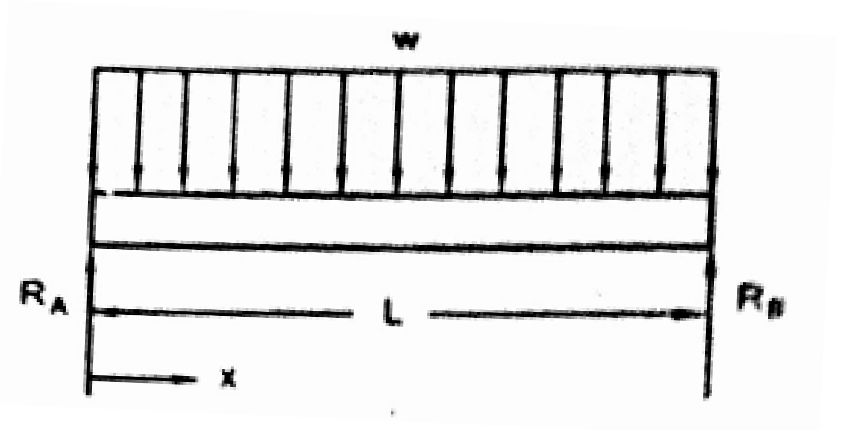


Shear and bending moment diagram

$$+\uparrow \sum F_y = 0$$

$$\text{or, } R_A + R_B - wL = 0$$

$$\text{or, } R_A + R_B = wL$$



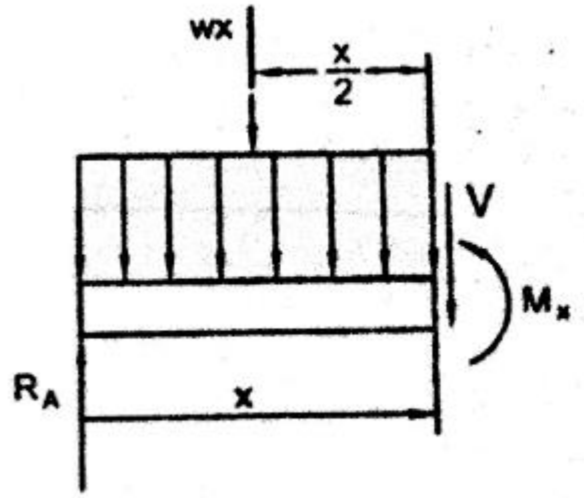
For the symmetric loading,

$$R_A = R_B = \frac{wL}{2}$$

To obtain the equation of shear force,

$$V_x = R_A - wx$$

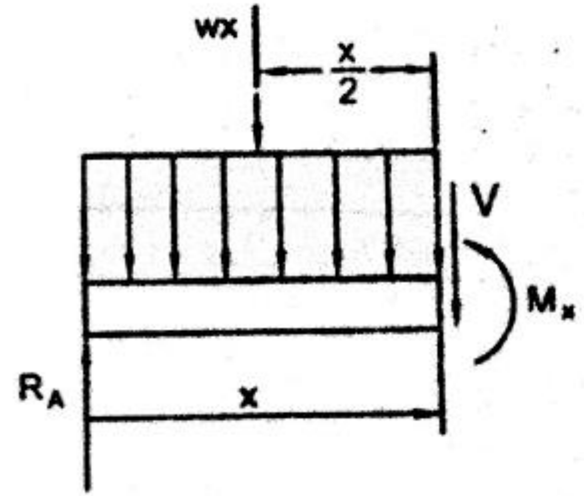
$$\text{or, } V_x = \frac{wL}{2} - wx$$



Shear and bending moment diagram

At $x = 0$, $\quad V_{x=0} = \frac{wL}{2}$

At $x = L$, $\quad V_{x=L} = -\frac{wL}{2}$



To find the location of zero shear,

$$\frac{wL}{2} - wx = 0$$

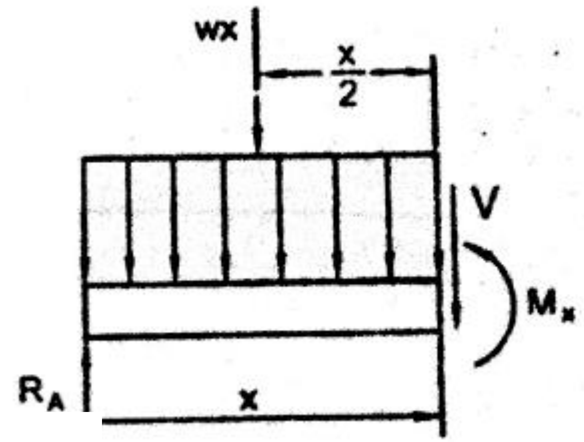
or, $x = \frac{L}{2}$

Shear and bending moment diagram

Bending moment,

For $0 < x < L$ $\qquad M_x = R_A x - wx\frac{x}{2}$

$$= \frac{wL}{2}x - \frac{wx^2}{2}$$



To find the location of maximum bending moment,

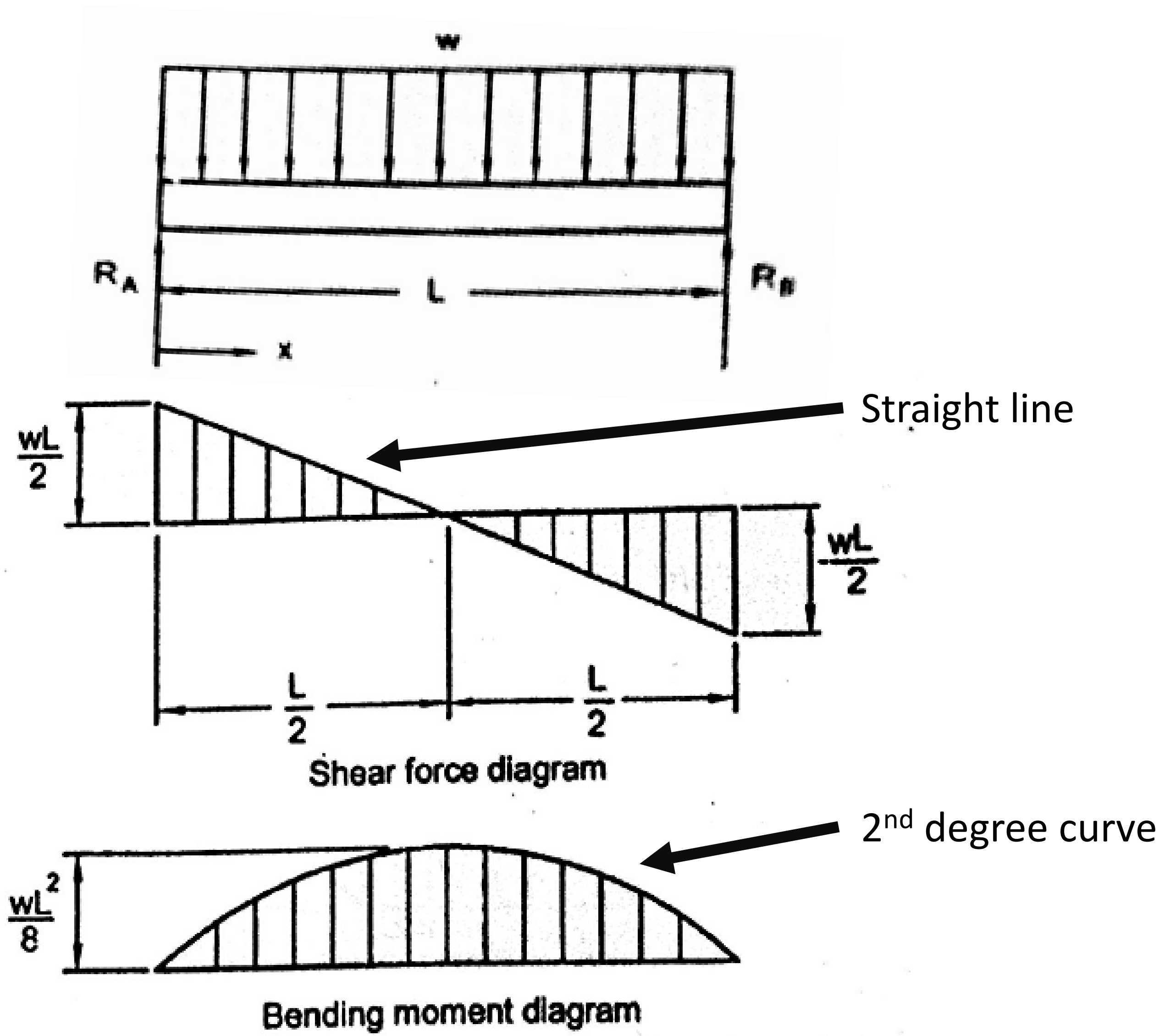
$$\frac{dM}{dx} = \frac{wL}{2} - wx = 0$$

$$\text{or, } x = \frac{L}{2}$$

$$M_{max} = \frac{wL}{2}\frac{L}{2} - \frac{w}{2}\frac{L^2}{4}$$

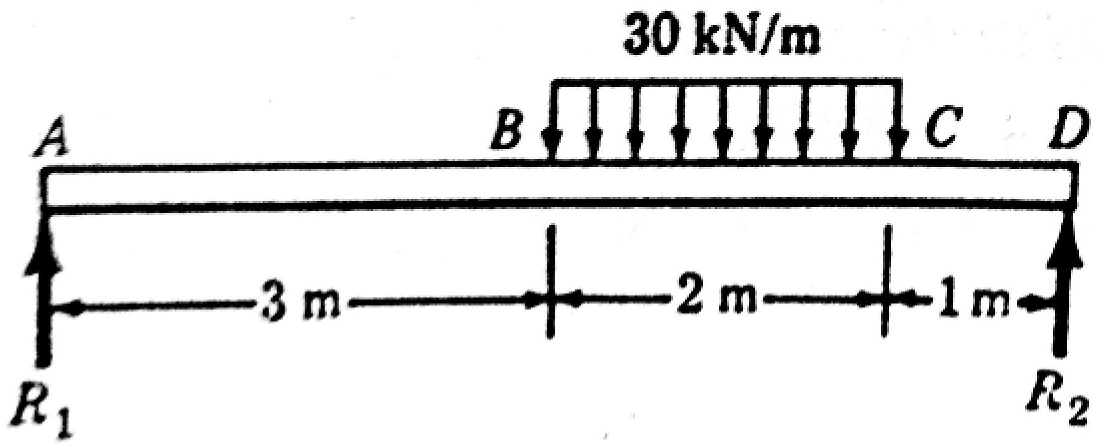
$$= \frac{wL^2}{8}$$

Shear and bending moment diagram



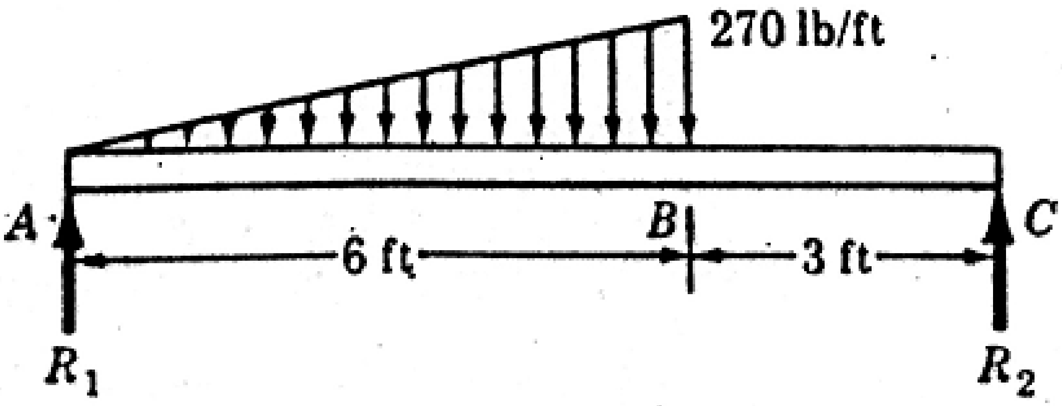
Problem#407(singer)

Draw the shear force and bending moment diagrams by using equations.
Also find the maximum bending moment and its location.



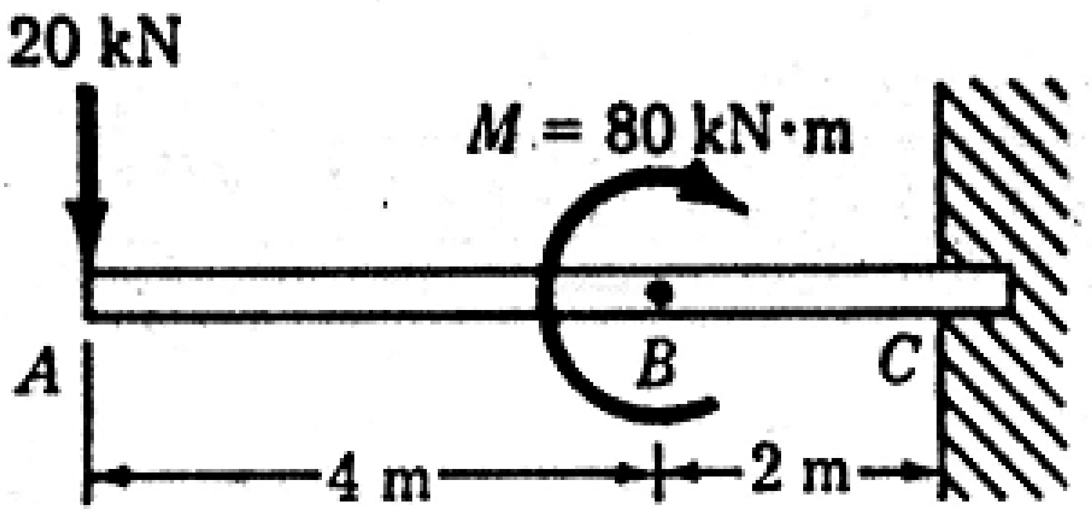
Problem# 419 (singer)

Draw the shear force and bending moment diagrams by using equations.
Also find the maximum bending moment and its location.



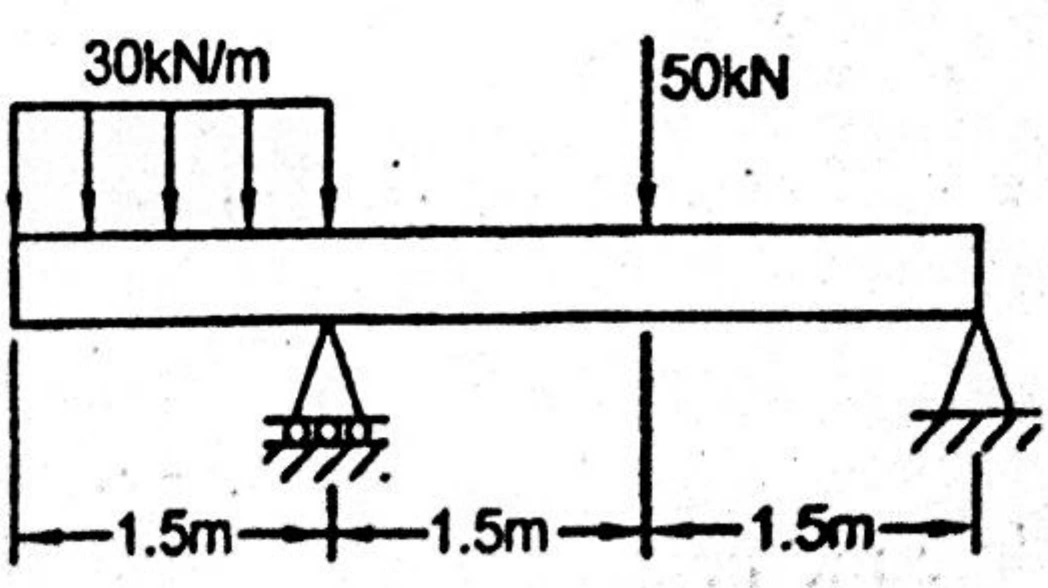
Problem# 418(singer)

Draw the shear force and bending moment diagrams by using equations.
Also find the maximum bending moment and its location.

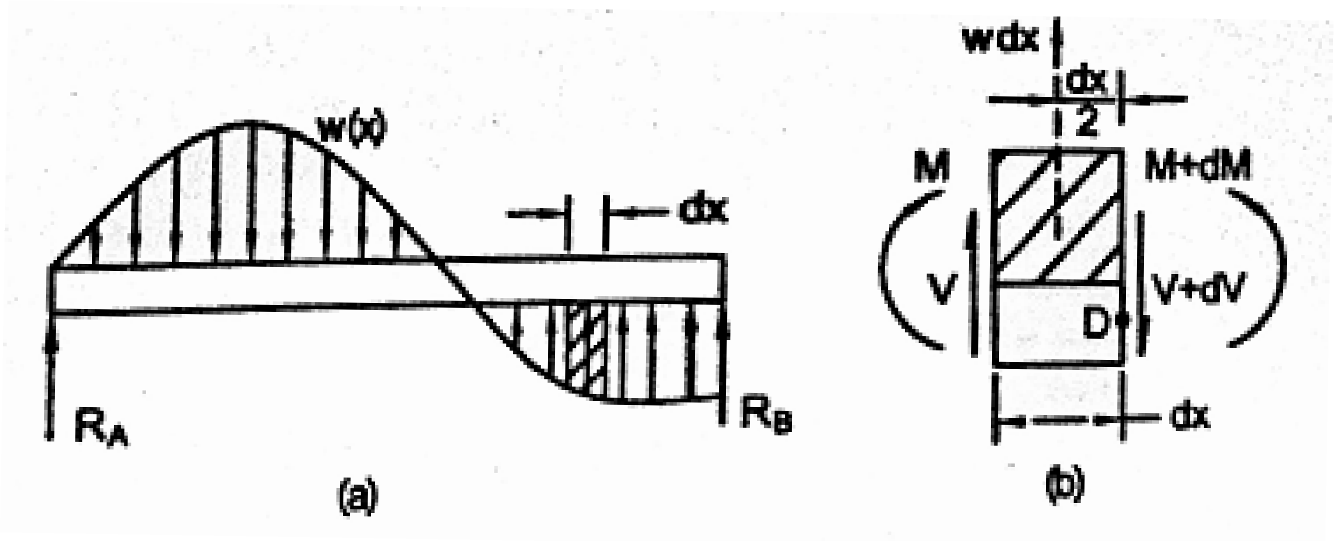


Problem#3.42(quamrul)

Draw the shear force and bending moment diagrams by using equations.
Also find the maximum bending moment and its location.



Relations among load, shear and moment



$$\Sigma P_y = 0: \text{ or, } V + wdx - (V + dV) = 0$$

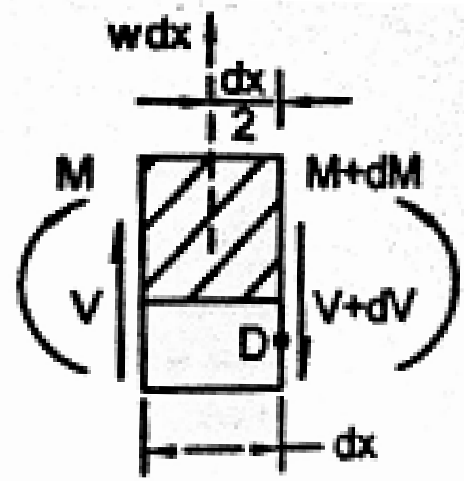
$$dV = wdx$$

$$\text{or, } w = \frac{dV}{dx} \qquad \text{......................(1)}$$

Relations among load, shear and moment

For the loading in the downward direction the relation of load and shear will be as,

$$-w = \frac{dV}{dx}$$



Now by summing the moments about a point D, one finds,

$$\Sigma M_D = 0 : \text{ or, } M + Vdx + (wdx)\frac{dx}{2} - (M + dM) = 0$$

Neglecting the square of the differential element compared to the other terms,

$$dM = Vdx$$

$$\text{or, } V = \frac{dM}{dx} \qquad \text{..............(2)}$$

Relations among load, shear and moment

Integrating (1), we get,

$$\int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w dx$$

$$\text{or, } V_2 - V_1 = \Delta V = (Area)_{load}$$

Integrating (2), we get,

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V dx$$

$$\text{or, } M_2 - M_1 = \Delta M = (Area)_{shear}$$

Again, we get,

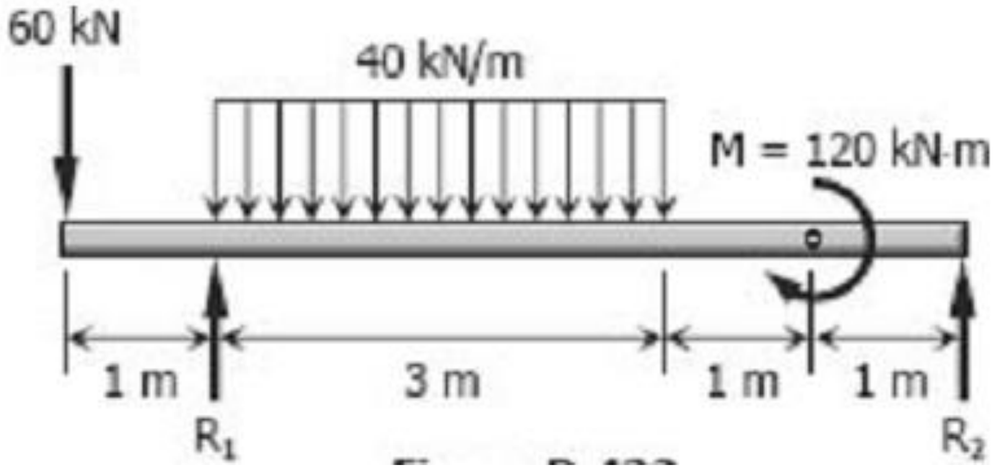
$$w = \frac{dV}{dx} = \text{Slope of shear diagram}$$

$$V = \frac{dM}{dx} = \text{Slope of moment diagram}$$

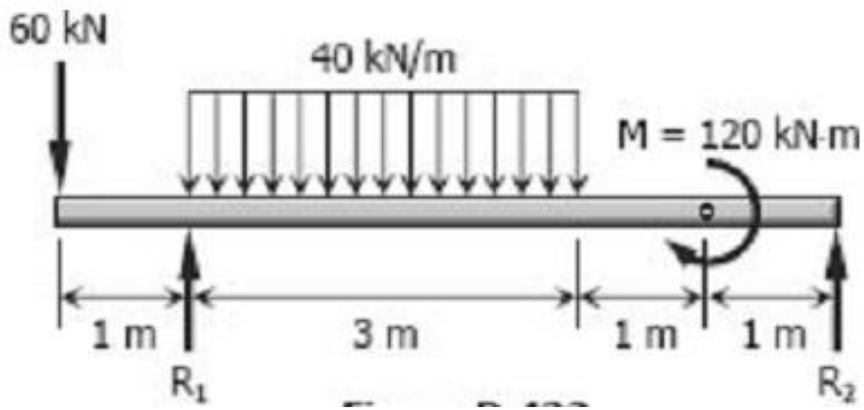
These relations among the load, shear and bending moment are useful in constructing shear force and bending moment diagrams.

Problem: 432 (singer)

Draw the shear force and bending moment diagrams by using relationships among load, shear and moment . Also find the maximum bending moment and its location.



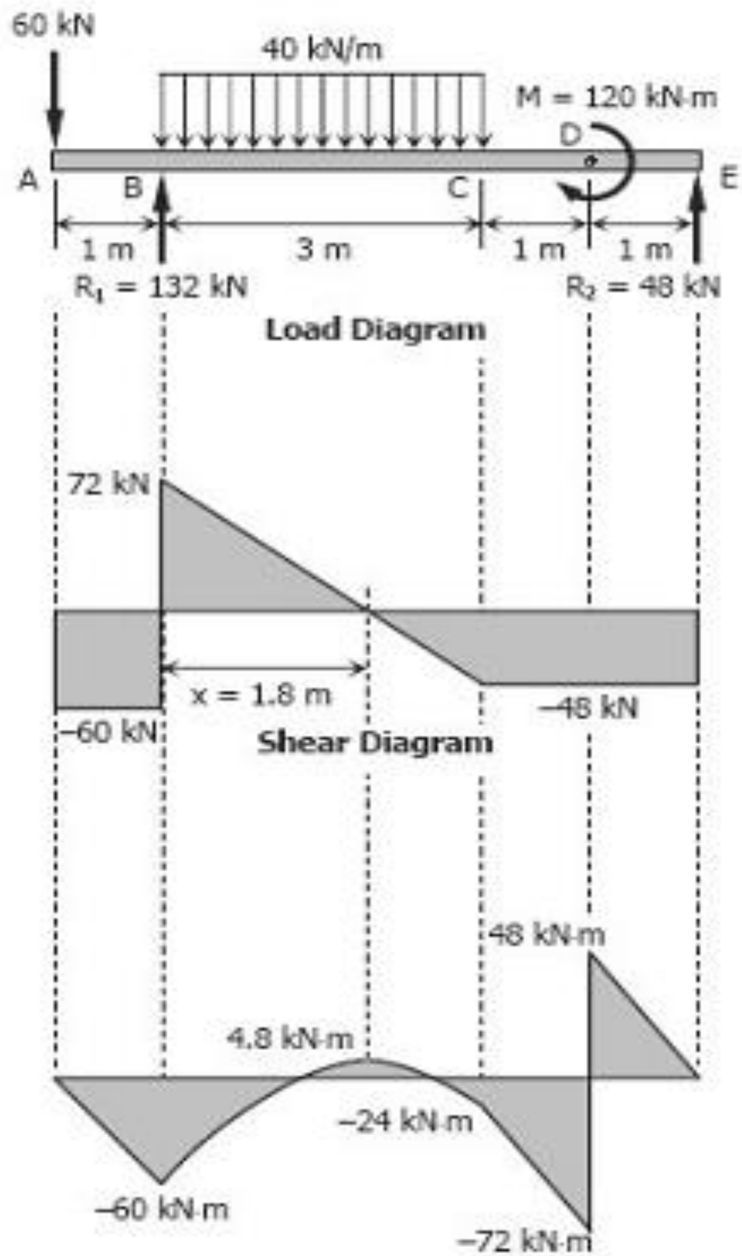
Solution



$$\Sigma M_E = 0$$
$$5R_1 + 120 = 6(60) + 40(3)(3.5)$$
$$R_1 = 132 \text{ kN}$$

$$\Sigma M_B = 0$$
$$5R_2 + 60(1) = 40(3)(1.5) + 120$$
$$R_2 = 48 \text{ kN}$$

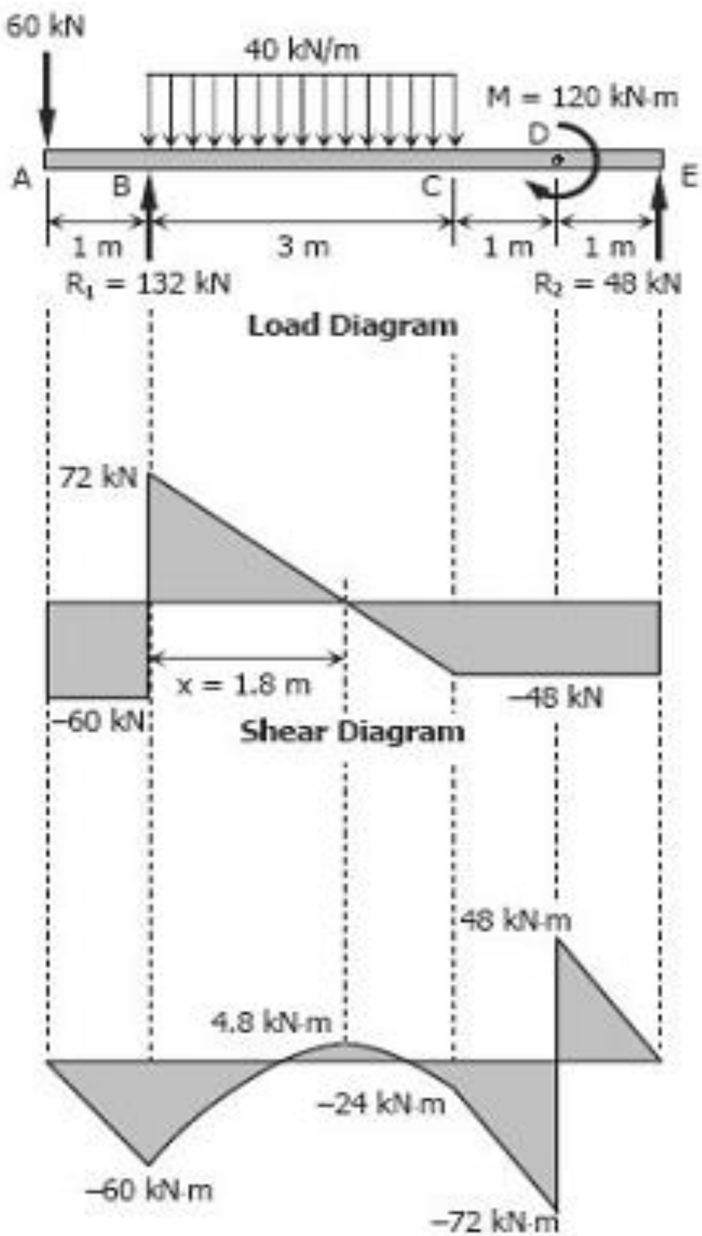
Solution:



To draw shear force diagram:

1. $V_A = -60$ kN
2. $V_B = V_A$ + Area in load diagram
   $V_B = -60 + 0 = -60$ kN
   $V_{B2} = V_B + R_1 = -60 + 132 = 72$ kN
3. $V_C = V_{B2}$ + Area in load diagram
   $V_C = 72 - 3(40) = -48$ kN
4. $V_D = V_C$ + Area in load diagram
   $V_D = -48 + 0 = -48$ kN
5. $V_E = V_D$ + Area in load diagram
   $V_E = -48 + 0 = -48$ kN
   $V_{E2} = V_E + R_2 = -48 + 48 = 0$
6. Solving for x:
   $x / 72 = (3 - x) / 48$
   $48x = 216 - 72x$
   $x = 1.8$ m

Solution



To draw bending moment diagram

1. $M_A = 0$
2. $M_B = M_A$ + Area in shear diagram

   $M_B = 0 - 60(1) = -60$ kN·m
3. $M_X = M_B$ + Area in shear diagram

   $M_X = -60 + ½ (1.8)(72) = 4.8$ kN·m
4. $M_C = M_X$ + Area in shear diagram

   $M_C = 4.8 - ½ (3 - 1.8)(48) = -24$ kN·m
5. $M_D = M_C$ + Area in shear diagram

   $M_D = -24 - ½ (24 + 72)(1) = -72$ kN·m

   $M_{D2} = -72 + 120 = 48$ kN·m
6. $M_E = M_{D2}$ + Area in shear diagram

   $M_E = 48 - 48(1) = 0$